Semi-inner products

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In studying the theory of Hilbert spaces, the inner product structure plays a significantly larger role than the norm; thus a Hilbert space acts more like an inner product space, than a particular Banach space. In order to carry these theories over to Banach spaces, there is a need for an inner product like structure.

The theory of semi-inner products were introduced by G. Lumer (1960) and J.R. Giles (1967). Like inner products, semi-inner products are positive definite and linear in the first argument. However, they are not necessarily symmetric. It is remarked that any normed space can be equipped by a semi-inner product. In the 1970s, P. M. Miličić provided a construction for a semi-inner product, using the Gâteaux lateral derivatives of the norm. Using semi-inner products, geometrical properties such as orthogonality and angles, may be introduced in normed spaces.

The lecture series will be organised as follows:

**Lecture 1:** Introduction to semi-inner product spaces (sips).

**Lecture 2:** Orthogonality in sips.

**Lecture 3:** Riesz representation theorem in sips.

**Lectures 4 and 5:** Angles in sips, and angular equivalence of normed spaces.